My guilty pleasures.

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Congruences of local origin

The original Ramanujan congruence:

$$\tau(n) \equiv \sigma_{11}(n) \bmod 691$$

follows from the fact that $\text{ord}_{691}\left(\frac{\zeta(12)}{\pi^{12}}\right) > 0$.

Question: Can we generate congruences between level 1 and level p?

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Dummigan, D.

Let $k \geq 4$ be even and p be prime. If $l > 3$ is a prime such that $\text{ord}_l\left((p^k - 1)\frac{B_k}{2k}\right) > 0$ then there exists a normalized eigenform $f \in S_k(\Gamma_0(p))$ with Hecke eigenvalues a_n satisfying:

$$a_q \equiv 1 + q^{k-1} \bmod \lambda \qquad \text{for } q \neq p$$

where $\lambda \mid l$ in $\mathbb{Q}_f$.

We can generate infinite families of such congruences if $p = 2$ and l is a Mersenne prime $M_{p_0} = 2^{p_0} - 1$.

Dummigan, D.

Let $p_0 > 2$ be prime and pick any m such that $\frac{M_{p_0}-1}{2p_0} \nmid m$. Then there exists $f \in S_{2mp_0}(\Gamma_0(2))$ with:

$$a_q \equiv 1 + q^{2mp_0 - 1} \bmod \lambda \qquad \text{for } q \neq p,$$

where $\lambda \mid M_{p_0}$ in $\mathbb{Q}_f$.

This is good computationally...if $p_0 > 3$ we can always take $m = 1$. Also the weight/level is small relative to the modulus.

Ask me for examples or for details of the paper!

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